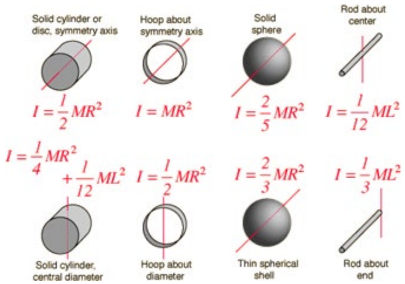
Rotational Inertia

Definitions

Rotational Inertia: The rotational inertia or moment of inertia, I, describes how the mass is arranged around the center of the rotation.

$I = \Sigma m r^2$	Moment of inertia	I	Kg m^2
	mass	m	kg
	Radius from pivot	r	m

The moment of inertia is different for different shapes.

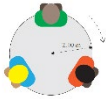


These variations **do not need to be remembered** – just the idea that they all (pretty much) depend upon r^2.

Equations

$\tau = I\alpha$	Torque	τ	N m
	Moment of inertia	I	Kg m^2
	Angular acceleration	α	rad s^{-2}
$E_{K(ROT)} = \frac{1}{2}I\omega^2$	Kinetic Energy	E_k	J
	Moment of inertia	I	Kg m^2
	Angular velocity	ω	rad s^{-1}
$\Delta E_p = mg\Delta h$	Change in Potential Energy	ΔE_p	J
	mass	m	kg
	Acceleration due to gravity	g	m s^{-2}
	Change in height	Δh	m
$E_{K(LIN)} = \frac{1}{2}mv^2$	Kinetic Energy	E_k	J
	mass	m	kg
	velocity	v	m s^{-1}

Questions

Mechanics 2019: QUESTION TWO

Three children are playing on a merry-go-round with a rotational inertia of 271 kg m^2. Once the children get the merry-go-round spinning, they stand evenly spaced around the outer edge. Each child has a mass of 28.0 kg, and the merry-go-round has a radius of 2.10 m.

(a) Assuming the rotational inertia of a child's mass on the edge of the disc is given by $I = mr^2$, show that the rotational inertia of the system is 641 kg m^2.

(b) The total energy of the system is 388 J. Show that:
(i) the angular velocity of the system is 1.10 rad s^{-1}, and
(ii) the linear velocity of one of the children is 2.31 m s^{-1}.

(c) One child drags her foot on the ground to bring the merry-go-round to a stop in 2.80 s. Calculate the amount of torque produced by the foot.

(d) The children get the merry-go-round spinning once again at a constant angular speed. Then each child moves inward towards the centre of the merry-go-round. Using physics principles, explain the effect this has on the rotational energy of the system.

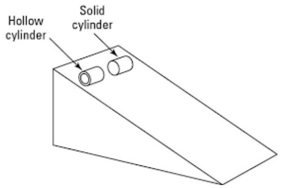
Terms

Conservation of Energy: When energy is transformed from one type of energy into another, the total energy before and after are always the same

Tips

The moment of inertia is the angular equivalent of mass.

An example of the effect of I:



If the two cylinders have the same mass, the solid cylinder will arrive at the bottom first due to the **conservation of energy**. The cylinders both start with the same gravitational potential energy. More energy is converted to rotational kinetic energy so less is left for linear kinetic energy.

Answers

(a) Total $I = I_{MGR} + I_{children} = 271 + (3 \times 28.0 \times 2.10^2) = 641.44$ kg m^2

(b) At max velocity $E_k(rot) = ½ I\omega^2 = 388$ J, $388 = ½\ 641\ \omega^2$, $\omega_{max} = 1.10$ rad s^{-1} so $v = \omega r = 1.10 \times 2.10 = 2.31$ m s^{-1}

(c)
$$\alpha = \frac{\Delta\omega}{t} = \frac{0 - 1.10 \text{ rad s}^{-1}}{2.80 \text{ s}} = 0.393 \text{ rad s}^{-2}$$
$$\tau = I\alpha = 641.44 \text{ kg m}^2 \times 0.393 \text{ rad s}^{-2} = 252 \text{ N m}$$

(d) As the children move inward the mass distribution decreases, thus rotational inertia decreases. kg m^2 s^{-1}. States that angular momentum is conserved because it is a closed system or because the external torques sum to zero. Assuming angular momentum is conserved, a decrease in rotational inertia results in a proportional increase in angular velocity. E_K-rotational = ½ $I\omega^2$, even though I decreases proportionally, because ω is squared, the rotational kinetic energy increases overall.