Simple Harmonic Motion Resonance

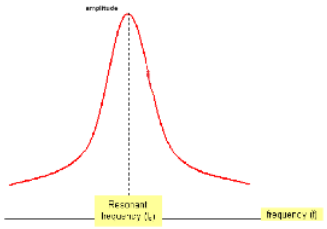
Definitions

All systems have their own natural (resonant) frequency. Specific examples with formulas that you are given:

- The simple pendulum
- The spring-mass system

(Although there are many more physical systems with resonant formula)

Because all systems have their own natural frequency, if you apply a driving force of the same frequency and in phase with the initial oscillations then resonance results, the amplitude of the oscillations gets larger and larger.



Equations

$T = 2\pi\sqrt{\frac{l}{g}}$	Time period	T	s
	length	l	m
	Acceleration due to gravity	g	m s^{-2}
$T = 2\pi\sqrt{\frac{m}{k}}$	Time period	T	s
	mass	m	kg
	Spring constant	k	N m^{-1}
$\omega = 2\pi f$	Angular velocity	ω	rad s^{-1}
	Frequency	f	Hz
$f = \frac{1}{T}$	Frequency	f	Hz
	Time period	T	s

Questions

LUCY ON HER SWING (2006;1)

Strength of gravity = 9.81 N kg^{-1}

Little Lucy loves playing on her homemade swing. It is a seat attached to a tree by a rope, and her mum pushes her to make her swing. The swing acts like a simple pendulum and Lucy's mum pushes her gently so that her swinging motion can always be considered to be simple harmonic motion. The angular frequency of Lucy's simple harmonic motion is 2.2 rad s^{-1}.

(a) Using the information given above, calculate the frequency of Lucy's motion.
(b) The period of Lucy's simple harmonic motion can be calculated to be 2.9 s. Calculate the length of the rope used to make the swing.
(c) Lucy's mum knows that, as Lucy grows, the length of the rope will have to be shortened. If the length of the rope is halved, explain what effect this would have on the period of Lucy's swings.

Terms

Pendulum: A mass (called "Bob") tied to a piece of string
Resonant frequency: The frequency of vibration of an elastic object that depends on the size, composition, and shape of the object

Tips

$T = \frac{2\pi}{\omega}$	Time period	T	s
	Angular velocity	ω	rad s^{-1}

Answers

(a) $\omega = 2\pi f \Rightarrow f = 2.2/2\pi = 0.35014 = 0.35$ Hz

(b) $T = 2\pi\sqrt{\frac{l}{g}}$ so $2.9 = 2 \times \pi \times \sqrt{\left(\frac{l}{9.81}\right)}$

$\Rightarrow l = 2.08980 = 2.1$ m

(c) As the length is shortened, *T* will also decrease. If *L* is halved, *T* will decrease by square root of 2. T = 2.056 s